The correction of hot-wire readings for proximity to a solid boundary

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When using a hot wire for velocity measurements close to a solid boundary, errors may be introduced if the effect of the boundary on the rate of heat loss from the wire is ignored. An experimental determination of the effect is described, in which a hot wire was mounted at various distances from a metal surface forming one wall of a two-dimensional channel. The rate of heat loss was determined electrically, and the air velocity at the wire found from the known laminar velocity profile. The application of the results to turbulent flows is discussed briefly.

1. Introduction

Apart from the practical difficulties involved in its manufacture and use, the hot-wire anemometer is the most versatile instrument yet devised for the measurement of mean and fluctuating velocities. Its principal advantages lie in the small size of the sensing element, its good response to high-frequency fluctuations and its suitability for electronic instrumentation. These features enable a practical instrument to be made which can, for instance, be used within a few thousandths of an inch of a solid surface, in regions of high velocity gradient, and to record fluctuating velocities of very high frequency. The writer was interested in measuring the velocity distribution very close to the surface in channel and boundary-layer flows, and it was considered desirable to investigate the effect of the solid boundary on the rate of heat loss from a hot wire.

The proximity of an infinite, flat, solid boundary having a much higher thermal conductivity than the fluid affects the heat loss from a wire by a modification of the temperature and velocity fields. The boundary is effectively at ambient air temperature, and so additional heat is extracted from the fluid that is heated by the wire, the effect increasing rapidly as the wire approaches the boundary. Many workers have noted this effect, and some have attempted to allow for it.

Piercy & Richardson (1928) measured the heat loss from a hot wire attached to a whirling arm rotating uniformly in still air. The whirling arm was co-axial with a fixed metal cylinder, and supported the hot wire close to the surface of the cylinder, and parallel to its axis. The experiments were later repeated by Piercy, Richardson & Winny (1956) and compared with an inviscid-flow analysis of the problem. The results of these investigations would not be expected to apply exactly to a hot wire in a boundary layer, because of the different velocity

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profiles involved in the two cases, and may not be sufficiently accurate as a correction.

Van der Hegge Zijnen (1924) obtained a form of correction by assuming that the effect of the wall could be represented by an additional heat loss independent of the stream velocity, and dependent only on the distance from the wall. The correction was obtained by measuring the total heat loss from the wire at a large distance from the wall, and at various closer distances from the wall, all the measurements being taken in still air. The 'extra' heat loss so obtained was then used to correct experimental velocity measurements. The same method was used by Dryden (1936) to correct hot-wire readings in a boundary layer. He found by experiment that the extra heat loss to the wall in still air was given by

$$H = \text{const.} \times l(\theta_w - \theta_a)^2/b,$$

where θ_w and θ_a are the wire and ambient temperatures, respectively, l the wire length and b its distance from the wall. The success of this method of correction depends on the assumption that the wire loses heat to the wall only by conduction, which is true only for very small wire Reynolds numbers R_w ($= u2a/\nu$, where a is the wire radius).

A probably more accurate method was that used by Reichardt (1940) who calibrated his wires close to the wall in a laminar-flow channel, and used the calibration in a channel with a turbulent flow having the same value of wall shear-stress. In view of the uncertainty about the form of correction to be applied, the present author decided to investigate the effect using Reichardt's method, and the rest of the report is concerned with this experimental investigation.

2. Experimental apparatus

Since it was desired to use the results of the investigation to correct hot-wire readings close to the wall in an incompressible boundary layer, a two-dimensional channel with fully developed laminar flow was thought to be eminently suitable, as the velocity profile close to the wall is similar to that in a laminar or turbulent boundary layer, and also the velocity in the channel is known accurately as a function of the pressure gradient along the channel and the distance from the wall. If these two and the total heat loss from the wire can be measured, the relation between heat loss, air velocity, and the distance from the wall can be deduced.

For small values of the distance y from the wall, the velocity u varies almost linearly with y, as in the same region of a laminar boundary layer, and in the viscous sublayer of a turbulent boundary layer. To achieve approximately the same relationship between u and y in the three cases, it is necessary to have the same value of U_τ ($= \sqrt{(\tau_0/\rho)}$, where τ_0 is the wall shear-stress) for each case. It was expected that values of U_τ up to 2 ft./sec would be found in the turbulent boundary layers to be investigated later, so the tests were designed to cover the range of U_τ from 0 to 2 ft./sec in the laminar-flow channel.

The need to cover this range of U_τ imposed a restriction on the width of the channel, since there is a channel Reynolds number above which turbulence starts to appear, and the velocity distribution ceases to be parabolic. In the channel used in these tests, bursts of turbulence were noticed at values of

$$R_c = \bar{U}h/\nu > 1360,$$

where $\bar{U}h$ is the volume flow per unit width of channel. The maximum width of channel to give a value of U_τ of 2 ft./sec without exceeding this Reynolds number is 0·085 in., and the velocity at the centre of the channel is then 45 ft./sec at the critical Reynolds number.

The channel used consisted of a glass plate and a duralumin plate, each 25 in. by 6 in., separated along their longer sides by 0·5 in.-wide steel shims of various thicknesses. The whole assembly could be clamped together along its length to give a parallel channel 25 in. long by 5 in. deep, with a gap-width variable over the range 0·025 in. to 0·300 in. With a 0·085 in. gap, the aspect ratio was 60 : 1. The necessary uniformity of such a small gap was obtained by the use of machine-ground shims, and by very careful machining of the duralumin plate. Static-pressure tappings along the centre line of this plate allowed the pressure gradient to be measured and its uniformity checked. Compressed air was supplied to the channel from a 4 in. flexible pipe and was filtered through a fine cotton screen before passing into the contraction at the inlet end of the channel. This filter removed the atmospheric dust which would otherwise have collected on the hot wire and affected its readings.

A length of 0·00015 in.-diameter tungsten wire was plated with copper in an acidified copper-sulphate solution until its diameter was about 0·005 in. A piece of the plated wire was then soldered across the ends of two steel sewing needles 0·020 in. in diameter mounted 0·187 in. apart in an insulating holder. The required length (about 0·1 in.) of tungsten wire was then exposed by the electro-chemical etching away of the copper with a small bubble of dilute sulphuric acid. Provided they were carefully mounted, these wires were kept straight by the springiness of the remaining copper-plated wire, and would last indefinitely unless broken by careless handling, their resistance remaining constant for several months.

The hot-wire probe was mounted on a traverse gear bolted firmly to the downstream end of the duralumin plate with the needles projecting 1 in. into the channel. The arrangement allowed the wire to be set parallel to the duralumin plate and normal to the flow, and traversed to within 0·001 in. of the plate, which formed the test wall. The distance of the wire from the wall could be read to an accuracy of 0·00005 in. on a micrometer head, and the zero distance was obtained to the same accuracy by viewing the wire and its reflexion in the test wall through a microscope and 45° mirror, the distance between the two images being measured on a graticule in the eye-piece.

3. Experiments in laminar flow

Several wires were heated in an oil-bath to determine their temperature coefficient of resistivity α_0. The mean value of α_0 was found to be 0·00338 per °C, and was sensibly constant over the range of interest, 0 to 150 °C. This value of

α_0 was used in the subsequent experiments to deduce the mean temperature of the etched portion of the wire from its measured resistance.

The hot wire was operated as one arm of a Wheatstone bridge, and the current adjusted before each reading to balance the bridge at a wire resistance corresponding to a mean temperature of 150 °C. Because of the high thermal conductivity of the relatively thick copper-plated ends, there is a variation of temperature along the hot wire, and the heat-loss readings were corrected to those for an infinite wire at the mean temperature of the actual wire, by a method given by King (1914) in his early paper on hot wires.

The correction is based on the assumptions (i) that the heat loss to the air per unit length from a small section of the wire is equal to that for an infinite wire, which implies that the aspect ratio of the wire must be large, and (ii) that the ends of the wire are at ambient temperature, due to the high conductivity of the supports. These assumptions lead to a hyperbolic cosine distribution of temperature along the wire, and hence a corrected Nusselt number. The Nusselt number is defined as $Nu = H/\pi kl(\theta_w - \theta_a)$, where H = heat loss from wire, k = conductivity of fluid.

Dimensional analysis for a wire of infinite length shows that

$$Nu = f(R_w, Pr, Gr, M, b/a), \tag{1}$$

provided the fluid properties can be assumed to be independent of temperature, and provided radiation from the wire is negligible, as it will be at such low temperatures. Here Pr is the Prandtl number and Gr the Grashof number ($= ga^3(\theta_w - \theta_a)/\nu^2\theta_a$ for a gas). For air at normal temperatures and low Mach numbers, the Prandtl number is constant and the Mach-number effect negligible. Collis & Williams (1959) have shown that free convection is important only if $Gr > R_w^3$. Thus for these wires, except for $R_w < 0{\cdot}005$, free convection is negligible, so that Nu is a function only of R_w and b/a. The results are plotted to show this relation. During preliminary tests it was found that the variation of fluid properties with temperature was important, since the results taken with the wire at different temperatures could not be plotted on one curve of Nu vs. R_w. The solution adopted was to evaluate all fluid properties at the mean of the wire and ambient temperatures, which enabled results taken at different temperatures to be closely correlated, both near to and far from the wall. This procedure was used by Collis & Williams, and has been adopted in presenting the final results.

For a range of values of b/a, the rate of heat loss (i^2r) from the wire at a constant temperature of 150 °C was measured for a suitable range of velocities. The 'free' calibration was also obtained, with the wire in the centre of the 0·085 in. channel, giving a value of b/a of about 500, sufficiently large for the effect of the wall to be negligible.

4. Presentation of results

The original results were cross-plotted to eliminate small errors, due mainly to inaccurate measurement of the distance from the wall at low values of b/a, giving the curves shown in figure 1. The most striking result to emerge from the tests is that, for $R_w > 0{\cdot}1$ and up to the highest values that could conveniently be measured, the effect of the wall is represented, for a particular wall distance, by a constant 'extra' heat loss, i.e. the curves are merely displaced upwards. This is shown to better advantage in figure 2, where $Nu(\theta_w/\theta_a)^{-0{\cdot}17}$ is plotted

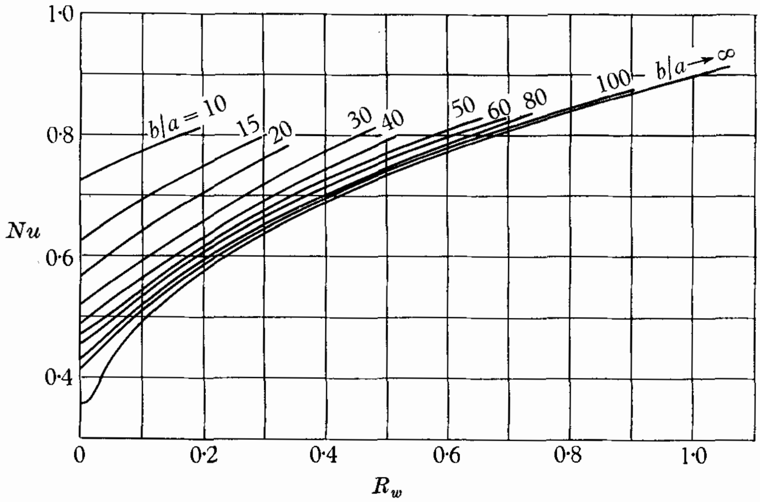

FIGURE 1. Heat loss from wire close to plane boundary.

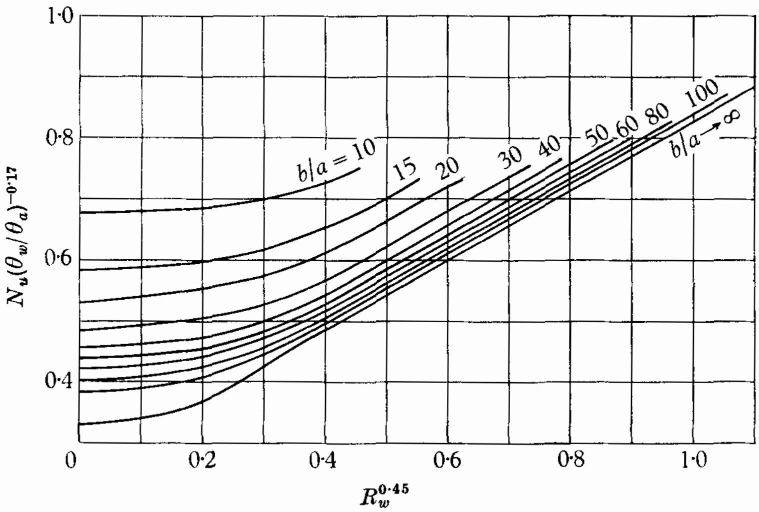

FIGURE 2. Results of figure 1 plotted according to equation (2).

against $R_w^{0{\cdot}45}$ to give, for R_w greater than about $0{\cdot}1$, a series of parallel straight lines having the equations

$$Nu(\theta_w/\theta_a)^{-0{\cdot}17} = A + 0{\cdot}56R_w^{0{\cdot}45}. \qquad (2)$$

This formula was obtained by Collis & Williams from an extensive series of measurements of two-dimensional convection from hot wires far from a solid boundary at similar Reynolds numbers and was found to fit the present author's results very accurately. Collis & Williams found A to be $0{\cdot}24$, but in these tests

A varies with b/a, tending to 0·26 as $b/a \to \infty$, and the variation is shown in figure 3. To provide a comparison with Van der Hegge Zijnen's 'wind-off' correction, figure 3 also shows the variation of $Nu_0(\theta_w/\theta_a)^{-0\cdot17}$ with b/a, where Nu_0 is the measured Nusselt number at zero speed. The value of $Nu_0(\theta_w/\theta_a)^{-0\cdot17}$ for $b/a \to \infty$ is 0·33, compared with 0·26 for A. Accordingly, the values of $Nu_0(\theta_w/\theta_a)^{-0\cdot17}$ have been reduced by 0·07, to match the curves at $b/a \to \infty$. The difference between the two curves then represents the error involved if Van der Hegge Zijnen's correction is used.

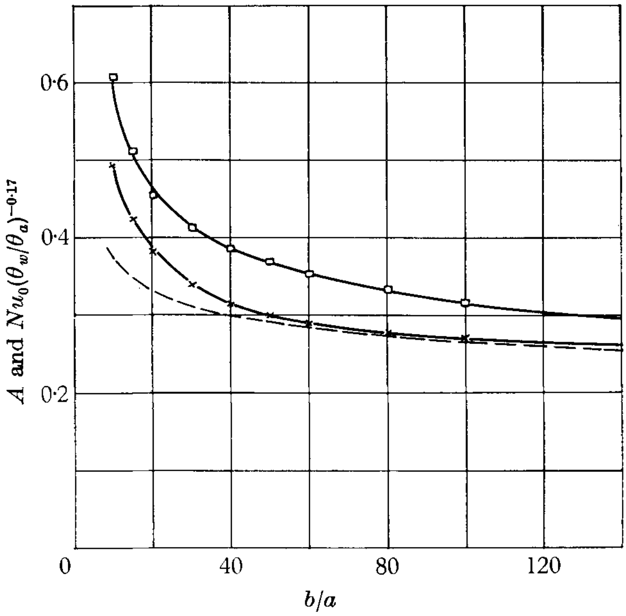

FIGURE 3. Comparison of wall-effect corrections. □, Values of A in equation (2); ×, measured $Nu_0(\theta_w/\theta_a)^{-0\cdot17}$; ----, theoretical $Nu_0(\theta_w/\theta_a)^{-0\cdot17}$.

The theoretical Nu_0 curve was obtained by J. E. Ff. Williams and the author as a solution of Poisson's equation, representing the wire and its image by a distribution of heat sources and sinks, respectively, having the required strength distribution to give the known temperature distribution along the wire. The conduction heat loss is given by

$$Nu_0 = 2\left(\frac{\cosh pl/2a}{\cosh pl/2a - 1}\right) \Big/ \int_0^{l/2a} \left(1 - \frac{\cosh px/a}{\cosh pl/2a}\right) \times \left[\frac{1}{\{1+(x/a)^2\}^{\frac{1}{2}}} - \frac{1}{\{1+4(b/a)^2+(x/a)^2\}^{\frac{1}{2}}}\right] d(x/a), \tag{3}$$

where $p^2 = Nu_0 k/K(1+a_w^*)$, K is the conductivity of the wire material, and a_w^* is the value of $\alpha_0(\theta_w - \theta_a)$ for an infinite wire. As the variation in Nu_0 is comparatively small, it was possible to obtain an approximate solution by assuming p constant. The integral was then evaluated numerically.

It thus seems that for wire Reynolds numbers greater than 0·1, an accurate correction for the effect of the wall at any value of b/a may be obtained by subtracting the appropriate value of $(A-0{\cdot}26)$ from the measured value of $Nu(\theta_w/\theta_a)^{-0{\cdot}17}$. It must be emphasized, however, that the Reynolds number in these experiments was limited as shown in figure 1, and although Collis & Williams have shown that the b/a curve of figure 2 remains linear up to $R_w = 44$, the simple correction given may not be accurate up to such high Reynolds numbers.

5. An illustrative example

The results in the form given above are not directly applicable to the correction of measured velocities close to a wall, but if the velocities fall in the linear range of figure 2, a reasonably accurate correction can be achieved with considerable

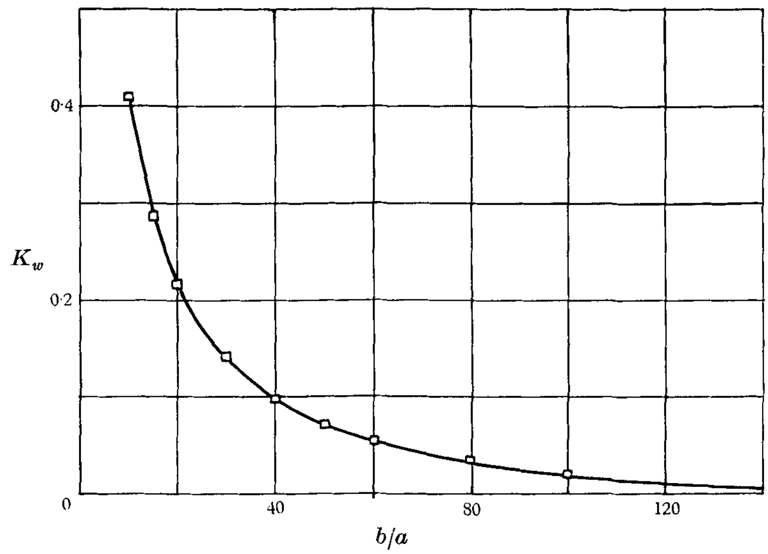

FIGURE 4. Correction to measured values of $R_w^{0{\cdot}45}$ for proximity to wall.

simplification. The correction in this case is the subtraction of a constant, K_w, from the value of $R_w^{0{\cdot}45}$, the constant depending on the value of b/a. The variation of K_w with b/a is shown in figure 4. Provided the variation of the value of ν used in calculating R_w with velocity u is small (it will be zero if the wire temperature is kept constant), $R_w \propto u$, and a graph similar to figure 4 can be constructed, relating the correction in $u^{0{\cdot}45}$ to b/a. For instance, with a wire of 0·00015 in. diameter, a wire temperature of 150 °C and an ambient temperature of 20 °C,

$$u^{0{\cdot}45} = 3{\cdot}73 R_w^{0{\cdot}45}\,(\text{ft./sec})^{0{\cdot}45}.$$

The correction to $u^{0{\cdot}45}$ at a value of b/a of, say, 50, is 0·26 and the measured value of $u^{0{\cdot}45}$ at a value of b/a of 50 must be reduced by this amount to give the correct velocity.

6. Application to turbulent flow

The results of this investigation were intended to serve the purpose of correcting hot-wire readings close to the wall in turbulent flow, in order to obtain the true mean velocity in these regions. Tests were subsequently performed to see whether the laminar-flow correction could in fact be applied to turbulent flows. These tests were performed in the same channel as the laminar-flow tests, but with a 0·25 in. gap to give turbulent flow with the same values of U_τ and hence the same velocity gradient at the wall as in the laminar case.

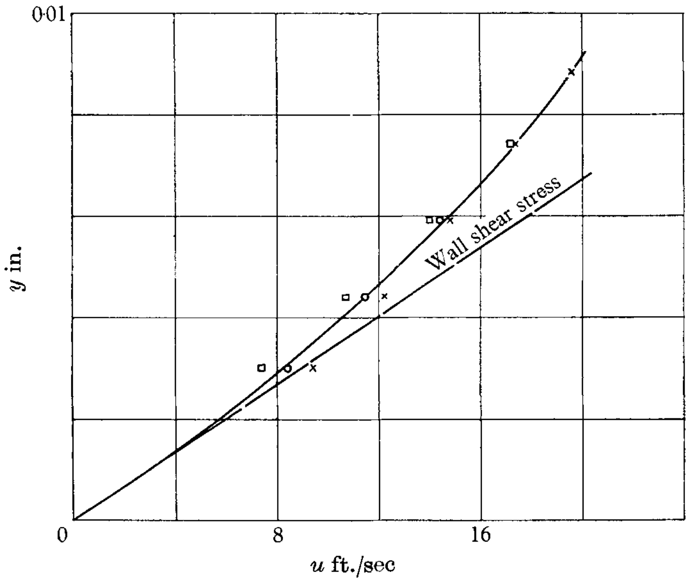

FIGURE 5. Measured mean-velocity profile close to surface in turbulent flow. ×, Measured velocity; □, with laminar-flow correction; ○, with half laminar-flow correction.

Velocities were measured with a constant-temperature hot-wire anemometer which used the non-linear characteristics of a triode valve to match the non-linearity of the hot-wire response and achieve an output linear with velocity. The instrument thus responded to the true mean velocity of flow far from the wall, and if the laminar-flow correction could be applied to the instantaneous velocity, the measured velocities could be corrected by a constant amount corresponding to the value of A for a particular distance from the wall.

A typical set of velocity measurements is shown in figure 5, together with the results obtained by applying the laminar-flow correction for proximity of the wall. Close to the surface, the true velocity profile must tend to the straight line shown, since this gives the wall shear-stress corresponding to the measured pressure gradient. Also, the true velocity profile must give du/dy increasing monotonically as the wall is approached. These considerations show that the true correction in turbulent flow must be of the same sign as the laminar correction but smaller. It therefore seems reasonable to assume that the true correction is a constant proportion of the laminar correction, and to attempt to determine the value of the proportion from diagrams such as figure 5. This approach led to the value of $0 \cdot 5 \pm 0 \cdot 1$ of the laminar correction. Although it has only been possible to obtain this rough estimate of the correction for turbulent flow, it is probably sufficiently accurate for determining the mean velocity close to the wall in most turbulent flows. The correction would not be expected to apply outside the region where the laminar and turbulent velocity profiles are generally similar, i.e. the viscous sublayer, and no attempt was made to extend the measurements outside this region.

It has not been found possible to explain satisfactorily why the factor should be less than unity, except for the observation that the turbulent motion which extends into the region considered will convect heat on the average away from the wall to a greater extent than does the laminar motion, and that the total heat loss to the wall will be thereby reduced.

It is hoped that these results, while covering only a limited range of variables, may prove useful to those wishing to use hot wires close to a solid boundary.

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